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Can von Neumann's Theory Meet the Deutsch-Jozsa Algorithm?

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Abstract The theoretical formalism of the implementation of the Deutsch-Jozsa algorithm relies on von Neumann's theory. We try to investigate whether von Neumann's theory meet our physical world. We derive a proposition concerning a quantum expectation value under the assumption of the existence of the orientation of reference frames in N spin-1/2 systems ($1 \leq N < +\infty$). This assumption intuitively depictures our physical world. However, the quantum predictions within the formalism of von Neumann's projective measurement violate the proposition with a magnitude that grows exponentially with the number of particles. Therefore, von Neumann's theory cannot depicture our physical world with a violation factor that grows exponentially with the number of particles. Hence, von Neumann's theory cannot meet the Deutsch-Jozsa algorithm. We propose the solution of the problem. Our solution is equivalent to changing Planck's constant ($\hbar$) to new constant ($\hbar/\sqrt{2}$). It may be that a new type of the quantum theory early approaches Newton's theory in the macroscopic scale than the old quantum theory does so.

Keywords Quantum measurement theory · Quantum computer

1 Introduction

The quantum theory (cf. [1–6]) gives approximate and at times remarkably accurate numerical predictions. Much experimental data approximately fits to the quantum predictions for the past some 100 years. We do not doubt the correctness of the quantum theory. The quantum theory also says new science with respect to information theory. The science is called the quantum information theory [6]. Therefore, the quantum theory gives us very useful another theory in order to create new information science and to explain the handing of raw experimental data in our physical world.

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As for the foundations of the quantum theory, Leggett-type nonlocal variables theory [7] is experimentally investigated [8–10]. The experiments report that the quantum theory does not accept Leggett-type nonlocal variables interpretation. As for the applications of the quantum theory, there are several attempts to use single-photon two-qubit states for quantum computing. Oliveira *et al.* implement Deutsch's algorithm [11] with polarization and transverse spatial modes of the electromagnetic field as qubits [12]. Single-photon Bell states are prepared and measured [13]. Also the decoherence-free implementation of Deutsch's algorithm is reported by using such single-photon and by using two logical qubits [14]. More recently, a one-way based experimental implementation of Deutsch's algorithm is reported [15].

To date, the quantum theory seems successful physical theory and it looks no problem in order to use it experimentally. Several researches address [1] the mathematical formulation of the quantum theory. Especially, von Neumann's theory is accepted widely. It is desirable that the quantum theory is also mathematically successful because we predict unknown physical phenomena precisely. Sometimes such predictions are effective in the field of elementary particle physics. We endure much time in order to see the fact by using, for example, large-scale accelerator. Further, Rolf Landauer says that *Information is Physical* [6]. We cannot create any computer without physical phenomena. This fact motivates us to investigate the Hilbert space formalism of the quantum theory. Here we ask: Does von Neumann's theory depicture our physical world? Unfortunately, it is not so even in both the macroscopic scale and the microscopic scale. The theoretical formalism of the implementation of the Deutsch-Jozsa algorithm [11, 16] relies on von Neumann's theory. Therefore, we cannot implement the Deutsch-Jozsa algorithm by using von Neumann's theory.

In this paper, we try to investigate whether von Neumann's theory meet our physical world. We derive a proposition concerning a quantum expectation value under the assumption of the existence of the orientation of reference frames in N spin-$1/2$ systems ($1 \leq N < +\infty$). This assumption intuitively depictures our physical world. However, the quantum predictions within the formalism of von Neumann's projective measurement violate the proposition with a magnitude that grows exponentially with the number of particles. Therefore, von Neumann's theory cannot depicture our physical world with a violation factor that grows exponentially with the number of particles. Hence, von Neumann's theory cannot meet the Deutsch-Jozsa algorithm. We propose the solution of the problem. Our solution is equivalent to changing Planck's constant ($\hbar$) to new constant ($\hbar/\sqrt{2}$). It may be that a new type of the quantum theory early approaches Newton's theory in the macroscopic scale than the old quantum theory does so.

Our discussion is very important. The reason is that our discussion reveals that we need new physical theories in order to explain our physical world informationally, to create new information science, and to predict new unknown physical phenomena efficiently. What are new physical theories? We cannot answer it at this stage. However, we expect that our discussion in this paper could contribute to creating new physical theories in order to explain our physical world, to create new information science, and to predict new unknown physical phenomena efficiently.

Throughout this paper, we confine ourselves to the two-level (e.g., electron spin, photon polarizations, and so on) and the discrete eigenvalue case. The number of settings of measuring apparatuses is two (two-setting model). These assumptions are used in several experimental situations. What we need is multiparticle described by the quantum system in pure spin-$1/2$ states lying in the x-y plane (2^N-dimensional state).

This paper is organized as follows. In Sect. 2, we show that von Neumann's theory does not meet our physical world. In Sect. 3, we modify von Neumann's projective measurement theory. In Sect. 4, we propose a new type of the Deutsch-Jozsa algorithm along with our modification of von Neumann's measurement theory. Section 5 concludes this paper.

2 von Neumann's Theory Does Not Meet Our Physical World

Assume that we have a set of N spins $\frac{1}{2}$. Each of them is a spin-1/2 pure state lying in the x-y plane. Let us assume that one source of N uncorrelated spin-carrying particles emits them in a state, which can be described as a multi spin-1/2 pure uncorrelated state. Let us parameterize the settings of the jth observer with a unit vector $\vec{n}_j$ (its direction along which the spin component is measured) with $j = 1, \ldots, N$. One can introduce the 'projective' correlation function, which is the average of the product of the results of von Neumann's projective measurement

$$E_{\mathrm{PM}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \langle r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N)\rangle_{\mathrm{avg}}, \tag{1}$$

where r is the projective result. We assume the value of r is ± 1 (in $(\hbar/2)^N$ unit), which is obtained if the measurement directions are set at $\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N$.

Also one can introduce a quantum correlation function with the system in such a pure uncorrelated state

$$E_{\mathrm{QM}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \mathrm{tr}[\rho \vec{n}_1 \cdot \vec{\sigma} \otimes \vec{n}_2 \cdot \vec{\sigma} \otimes \cdots \otimes \vec{n}_N \cdot \vec{\sigma}] \tag{2}$$

where $\otimes$ denotes the tensor product, $\cdot$ the scalar product in $\mathbf{R}^2$, $\vec{\sigma} = (\sigma_x, \sigma_y)$ is a vector of two Pauli operators, and ρ is a pure uncorrelated state,

$$\rho = \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_N \tag{3}$$

with $\rho_j = |\Psi_j\rangle\langle\Psi_j|$ and $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the x-y plane.

One can write the observable (unit) vector $\vec{n}_j$ in a plane coordinate system as follows:

$$\vec{n}_j(\theta_j^{k_j}) = \cos\theta_j^{k_j} \vec{x}_j^{(1)} + \sin\theta_j^{k_j} \vec{x}_j^{(2)}, \tag{4}$$

where $\vec{x}_j^{(1)} = \vec{x}$ and $\vec{x}_j^{(2)} = \vec{y}$ are the Cartesian axes. Here, the angle $\theta_j^{k_j}$ takes two values (two-setting model):

$$\theta_j^1 = 0, \qquad \theta_j^2 = \frac{\pi}{2}. \tag{5}$$

We derive a necessary condition to be satisfied by the quantum correlation function with the system in a pure uncorrelated state given in (2). In more detail, we derive the value of the scalar product of the quantum correlation function, E_{QM} given in (2), i.e., $(E_{\mathrm{QM}}, E_{\mathrm{QM}})$. We use decomposition (4). We introduce simplified notations as

$$T_{i_1 i_2 \ldots i_N} = \mathrm{tr}[\rho \vec{x}_1^{(i_1)} \cdot \vec{\sigma} \otimes \vec{x}_2^{(i_2)} \cdot \vec{\sigma} \otimes \cdots \otimes \vec{x}_N^{(i_N)} \cdot \vec{\sigma}] \tag{6}$$

and

$$\vec{c}_j = (c_j^1, c_j^2) = (\cos\theta_j^{k_j}, \sin\theta_j^{k_j}). \tag{7}$$

Then, we have

$$
\begin{aligned}
(E_{\rm QM}, E_{\rm QM}) &= \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \left(\sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N} c_1^{i_1} \cdots c_N^{i_N} \right)^2 \\
&= \sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N}^2 \le 1,
\end{aligned} \tag{8}
$$

where we use the orthogonality relation $\sum_{k_j=1}^{2} c_j^{\alpha} c_j^{\beta} = \delta_{\alpha,\beta}$. The value of $\sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N}^2$ is bounded as $\sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N}^2 \le 1$. We have

$$
\prod_{j=1}^{N} \sum_{i_j=1}^{2} (\mathrm{tr}[\rho_j \vec{x}_j^{(i_j)} \cdot \vec{\sigma}])^2 \le 1. \tag{9}
$$

From the convex argument, all quantum separable states must satisfy the inequality (8). Therefore, it is a separability inequality. It is important that the separability inequality (8) is saturated iff ρ is a multi spin-1/2 pure uncorrelated state such that, for every j, $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the x-y plane. The reason of the inequality (8) is due to the following quantum inequality

$$
\sum_{i_j=1}^{2} (\mathrm{tr}[\rho_j \vec{x}_j^{(i_j)} \cdot \vec{\sigma}])^2 \le 1. \tag{10}
$$

The inequality (10) is saturated iff $\rho_j = |\Psi_j\rangle\langle\Psi_j|$ and $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the x-y plane. The inequality (8) is saturated iff the inequality (10) is saturated for every j.Thus we have the maximal possible value of the scalar product as a quantum proposition concerning our physical world

$$
(E_{\rm QM}, E_{\rm QM})_{\max} = 1 \tag{11}
$$

when the system is in such a multi spin-1/2 pure uncorrelated state.

On the other hand, a correlation function satisfies a projective measurement theory if it can be written as

$$
E_{\rm PM}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \lim_{m\to\infty} \frac{\sum_{l=1}^{m} r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l)}{m} \tag{12}
$$

where l denotes a label and r is the result of von Neumann's projective measurement of the dichotomic observables parameterized by directions of $\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N$.

Assume the quantum correlation function with the system in a pure uncorrelated state given in (2) admits a projective measurement theory. One has the following proposition concerning a projective measurement theory

$$
E_{\rm QM}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \lim_{m\to\infty} \frac{\sum_{l=1}^{m} r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l)}{m}. \tag{13}
$$

In what follows, we show that we cannot assign the truth value "1" for the proposition (13) concerning a projective measurement theory.

Assume the proposition (13) is true.

By changing the label l into l' and by changing the label m into m', we have same quantum expectation value as follows

$$E_{\mathrm{QM}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \lim_{m' \to \infty} \frac{\sum_{l'=1}^{m'} r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l')}{m'}. \tag{14}$$

An important note here is that the value of the right-hand-side of (13) is equal to the value of the right-hand-side of (14) because we only change the labels.

We abbreviate $r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l)$ to $r(l)$ and $r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l')$ to $r(l')$.

We have

$$\begin{aligned}
(E_{\mathrm{QM}}, E_{\mathrm{QM}}) &= \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \left(\lim_{m \to \infty} \frac{\sum_{l=1}^{m} r(l)}{m} \times \lim_{m' \to \infty} \frac{\sum_{l'=1}^{m'} r(l')}{m'} \right) \\
&= \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \left(\lim_{m \to \infty} \frac{\sum_{l=1}^{m}}{m} \cdot \lim_{m' \to \infty} \frac{\sum_{l'=1}^{m'}}{m'} r(l) r(l') \right) \\
&\le \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \left(\lim_{m \to \infty} \frac{\sum_{l=1}^{m}}{m} \cdot \lim_{m' \to \infty} \frac{\sum_{l'=1}^{m'}}{m'} |r(l) r(l')| \right) \\
&= \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \left(\lim_{m \to \infty} \frac{\sum_{l=1}^{m}}{m} \cdot \lim_{m' \to \infty} \frac{\sum_{l'=1}^{m'}}{m'} \right) = 2^N.
\end{aligned} \tag{15}$$

We use the following fact

$$|r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l) r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l')| = +1. \tag{16}$$

The inequality (15) is saturated since we have

$$\begin{aligned}
&\{l | r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l) = 1 \wedge l \in \mathbf{N}\} = \{l' | r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l') = 1 \wedge l' \in \mathbf{N}\}, \\
&\{l | r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l) = -1 \wedge l \in \mathbf{N}\} = \{l' | r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, l') = -1 \wedge l' \in \mathbf{N}\}.
\end{aligned} \tag{17}$$

Hence one has the following proposition concerning a projective measurement theory

$$(E_{\mathrm{QM}}, E_{\mathrm{QM}})_{\max} = 2^N. \tag{18}$$

Clearly, we cannot assign the truth value "1" for two propositions (11) (concerning our physical world) and (18) (concerning a projective measurement theory), simultaneously, when the system is in a multiparticle pure uncorrelated state. Of course, each of them is a spin-1/2 pure state lying in the x-y plane. Therefore, we are in the contradiction when the system is in such a multiparticle pure uncorrelated state. Thus, we cannot accept the validity of the proposition (13) (concerning a projective measurement theory) if we assign the truth value "1" for the proposition (11) (concerning our physical world). In other words, such projective measurement theory does not reveal our physical world.

3 Solution of the Problem of von Neumann's Theory

In this section, we solve the contradiction presented in the previous section. We have the maximal possible value of the scalar product as a quantum proposition concerning our physical world

$$(E_{\rm QM}, E_{\rm QM})_{\max} = 1 \tag{19}$$

when the system is in such a multi spin-1/2 pure uncorrelated state. On the other hand, one has the following proposition concerning a projective measurement theory

$$(E_{\rm QM}, E_{\rm QM})_{\max} = 2^N. \tag{20}$$

We cannot assign the truth value "1" for two propositions (19) (concerning our physical world) and (20) (concerning a projective measurement theory), simultaneously, when the system is in a multiparticle pure uncorrelated state. Of course, each of them is a spin-1/2 pure state lying in the x-y plane. Therefore, we are in the contradiction when the system is in such a multiparticle pure uncorrelated state.

We introduce the following hypothesis:

Hypothesis *We assume the value of r is $\pm\frac{1}{\sqrt{2^N}}$ (in $(\hbar/2)^N$ unit), which is obtained if the measurement directions are set at $\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N$.*

When we accept this hypothesis, the proposition (20) (concerning a projective measurement theory) becomes the following new proposition concerning a quantum measurement theory (two-setting model)

$$(E_{\rm QM}, E_{\rm QM})_{\max} = 1. \tag{21}$$

We can assign the truth value "1" for both two propositions (19) (concerning our physical world) and (21) (concerning a quantum measurement theory), simultaneously, when the system is in a multiparticle pure uncorrelated state. Of course, each of them is a spin-1/2 pure state lying in the x-y plane. Therefore, we are not in the contradiction when the system is in such a multiparticle pure uncorrelated state. Hence, we solve the contradiction presented in the previous section by changing the value of the result of quantum measurements. Our solution is equivalent to changing Planck's constant ($\hbar$) to new constant ($\hbar/\sqrt{2}$).

4 New Type of the Deutsch-Jozsa Algorithm

The earliest quantum algorithm, the Deutsch-Jozsa algorithm, is representative to show that quantum computation is faster than classical counterpart with a magnitude that grows exponentially with the number of qubits.

Let us follow the argumentation presented in [6].

The application, known as *Deutsch's problem*, may be described as the following game. Alice, in Amsterdam, selects a number x from 0 to $2^N - 1$, and mails it in a letter to Bob, in Boston. Bob calculates the value of some function $f : \{0, \ldots, 2^N - 1\} \to \{0, 1\}$ and replies with the result, which is either 0 or 1. Now, Bob has promised to use a function f which is of one of two kinds; either the value of $f(x)$ is constant for all values of x, or else the value of $f(x)$ is balanced, that is, equal to 1 for exactly half of all the possible x, and 0 for the other half. Alice's goal is to determine with certainty whether Bob has chosen a constant or a balanced function, corresponding with him as little as possible. How fast can she succeed?

In the classical case, Alice may only send Bob one value of x in each letter. At worst, Alice will need to query Bob at least $2^N/2 + 1$ times, since she may receive $2^N/2$ 0s before finally getting a 1, telling her that Bob's function is balanced. The best deterministic classical algorithm she can use therefore requires $2^N/2+1$ queries. Note that in each letter, Alice sends Bob N bits of information. Furthermore, in this example, physical distance is being used to artificially elevate the cost of calculating $f(x)$, but this is not needed in the general problem, where $f(x)$ may be inherently difficult to calculate.

If Bob and Alice were able to exchange qubits, instead of just classical bits, and if Bob agreed to calculate $f(x)$ using a unitary transformation U_f, then Alice could achieve her goal in just one correspondence with Bob, using the following algorithm.

Alice has an N qubit register to store her query in, and a single qubit register which she will give to Bob, to store the answer in. She begins by preparing both her query and answer registers in a superposition state. Bob will evaluate $f(x)$ using quantum parallelism and leave the result in the answer register. Alice then interferes states in the superposition using a Hadamard transformation (a unitary transformation), $H=(\sigma_x+\sigma_z)/\sqrt{2}$, on the query register, and finishes by performing a suitable measurement to determine whether f was constant or balanced.

Let us follow the quantum states through this algorithm. The input state is

$$|\psi_0\rangle = |0\rangle^{\otimes N}|1\rangle. \tag{22}$$

Here the query register describes the state of N qubits all prepared in the $|0\rangle$ state. After the Hadamard transformation on the query register and the Hadamard gate on the answer register we have

$$|\psi_1\rangle = \sum_{x\in\{0,1\}^N} \frac{|x\rangle}{\sqrt{2^N}}\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]. \tag{23}$$

The query register is now a superposition of all values, and the answer register is in an evenly weighted superposition of $|0\rangle$ and $|1\rangle$. Next, the function f is evaluated (by Bob) using $U_f: |x,y\rangle \to |x, y\oplus f(x)\rangle$, giving

$$|\psi_2\rangle = \pm\sum_x \frac{(-1)^{f(x)}|x\rangle}{\sqrt{2^N}}\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]. \tag{24}$$

Here $y\oplus f(x)$ is the bitwise XOR (exclusive OR) of y and $f(x)$. Alice now has a set of qubits in which the result of Bob's function evaluation is stored in the amplitude of the qubit superposition state. She now interferes terms in the superposition using a Hadamard transformation on the query register. To determine the result of the Hadamard transformation it helps to first calculate the effect of the Hadamard transformation on a state $|x\rangle$. By checking the cases $x=0$ and $x=1$ separately we see that for a single qubit $H|x\rangle = \sum_z(-1)^{xz}|z\rangle/\sqrt{2}$. Thus

$$H^{\otimes N}|x_1,\ldots,x_N\rangle = \frac{\sum_{z_1,\ldots,z_N}(-1)^{x_1z_1+\cdots+x_Nz_N}|z_1,\ldots,z_N\rangle}{\sqrt{2^N}}. \tag{25}$$

This can be summarized more succinctly in the very useful equation

$$H^{\otimes N}|x\rangle = \frac{\sum_z(-1)^{x\cdot z}|z\rangle}{\sqrt{2^N}}, \tag{26}$$

where $x \cdot z$ is the bitwise inner product of x and z, modulo 2. Using this equation and (24) we can now evaluate $|\psi_3\rangle$,

$$|\psi_3\rangle = \pm \sum_z \sum_x \frac{(-1)^{x\cdot z + f(x)}|z\rangle}{\sqrt{2^N}} \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]. \tag{27}$$

Alice now observes the query register. Note that the absolute value of the amplitude for the state $|0\rangle^{\otimes N}$ is $\sum_x (-1)^{f(x)}/2^N$. Let's look at the two possible cases—f constant and f balanced—to discern what happens. In the case where f is constant the absolute value of the amplitude for $|0\rangle^{\otimes N}$ is $+1$. Because $|\psi_3\rangle$ is of unit length it follows that all the other amplitudes must be zero, and an observation will yield $(+\frac{1}{\sqrt{2}})$s for all N qubits in the query register. Thus, global measurement outcome is $(+\frac{1}{\sqrt{2^N}})$. If f is balanced then the positive and negative contributions to the absolute value of the amplitude for $|0\rangle^{\otimes N}$ cancel, leaving an amplitude of zero, and a measurement must yield a result other than $+\frac{1}{\sqrt{2}}$ (i.e., $-\frac{1}{\sqrt{2}}$) on at least one qubit in the query register. Summarizing, if Alice measures all $(+\frac{1}{\sqrt{2}})$s and global measurement outcome is $(+\frac{1}{\sqrt{2^N}})$ the function is constant; otherwise the function is balanced.

We notice that the difference between $+\frac{1}{\sqrt{2^N}}$ and $-\frac{1}{\sqrt{2^N}}$ is approximately zero when $N \gg 1$. We question if the Deutsch-Jozsa algorithm in the macroscopic scale is possible or not. This question is open problem.

5 Conclusions

In conclusion, we have investigated whether von Neumann's theory meet our physical world. We have derived a proposition concerning a quantum expectation value under the assumption of the existence of the orientation of reference frames in N spin-1/2 systems ($1 \leq N < +\infty$). This assumption has intuitively depictured our physical world. However, the quantum predictions within the formalism of von Neumann's projective measurement have violated the proposition with a magnitude that grows exponentially with the number of particles. Therefore, von Neumann's theory cannot have depictured our physical world with a violation factor that grows exponentially with the number of particles. Hence, von Neumann's theory cannot have met the Deutsch-Jozsa algorithm. We have proposed the solution of the problem. Our solution has been equivalent to changing Planck's constant ($\hbar$) to new constant ($\hbar/\sqrt{2}$). It may have been that a new type of the quantum theory early approaches Newton's theory in the macroscopic scale than the old quantum theory does so.

What are new physical theories? We cannot answer it at this stage. However, we expect that our discussion in this paper could contribute to creating new physical theories in order to explain our physical world, to create new information science, and to predict new unknown physical phenomena efficiently.

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